Students ‘Holding’ the Moment: Learning Mathematics in an Inquiry Mathematics Classroom

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Teachers of inquiry mathematics face many challenges as understandings of how children learn in this environment are still developing. Children learn as they hold onto the moment of not knowing what to do. This paper operationalises learning for a small group of Year Three students exploring the usefulness of maps. As part of a PhD study using Design Research, the teacher-as-researcher identified perturbations generated by the students in her class and applied an operational definition of learning to track student development of mathematical knowledge.

The quest for knowing how students learn mathematics in primary classrooms results in continued research illustrating effective teaching practices and learning strategies, so much so that there may appear a range of well-informed practices found in any one classroom. Teaching and learning mathematics through inquiry is an approach which fosters skills to promote positive classroom behaviours such as collaboration through a community of practice (Goos, 2004) and critical thinking skills through cycles of reflection (Rusche & Jason, 2011; Allmond, Makar, & Wells, 2010). An inquiry classroom fosters the development of mathematical knowledge in students through the solution of problems that are ill-structured, as everyday problems involving mathematics are (Makar, 2007). In inquiry mathematics, students’ ideas and mathematical notions are often challenged as they are forced to put strategies and thinking into practice and to defend their choices amongst peers. Difficulty lies in representing a student’s broad understandings when assessing. A greater knowledge of how and what students learn in inquiry is required before the development of assessment practices can take place. Harel and Koichu (2010) provided an operational definition of learning which valued perturbations as part of the learning journey. This paper hopes to identify and describe learning in an inquiry classroom using an operational definition of learning when perturbations are generated by the students themselves, or students ‘hold’ the moment when they are unsure about how to continue on a task, showing the value in listening to students when teaching using this approach.

Data presented in this paper is from one particular Year Three (students aged 8-9) classroom where the inquiry focused on the features of a map which make it most effective. Learning will be described for three students, chosen because they worked collaboratively in this unit of work, and due to the nature of a discussion with the three students about challenges they encountered. More research on the benefits of teaching mathematics using inquiry is required in order to refine this pedagogical approach. This study contributes understandings about learning to a larger PhD study using Design Research (Cobb, et al. 2003) where the teacher-as-researcher aims to develop ways to assess student mathematical understanding in inquiry that aligns with how students learn in this pedagogical approach.

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Literature

Inquiry

It can be useful to contrast a traditional mathematics classroom to an inquiry mathematics classroom to better understand the difference in pedagogy. Although not advocating one approach over the other, an inquiry classroom values learning for understanding through discussion and collaboration in a climate of intellectual challenge (Goos, 2004). Mathematical inquiry presents problems to solve that are important to students. The consideration of relevance to the students' world subscribes to a constructivist approach which values an organic connection between education and personal experience to foster learning connections (Dewey, 1938). Teachers of inquiry tend to carefully select problems which are relevant to the students and that will present a 'roadblock' or problematic to the student. Students learn to be independently numerate when they try and sort out mathematical problems, when they 'hold' themselves in moments where they don't know what to do. These moments help students to develop their role as a learner (DET, 2004) where they will face real-life problems involving mathematics every day. This notion applies to inquiry where mathematical problems reflect the real world. An inquiry classroom is more student-centred valuing variety in how problems are solved as opposed to a traditional mathematics classroom which is teacher-centred and fosters learnt procedures.

A mathematical inquiry may be a 'messy', ill-structured problem or question reflecting a context which is similar to real-life problems. Makar (2007) explains how nearly all everyday problems in life can be ill-structured. In inquiry, teachers guide the students through multiple learning pathways based on the knowledge shared as a whole class. Questions which are more ill-structured 'set up' complications for students as there can be a large number of open constraints (Makar, 2007) especially when words such as 'best' or 'biggest' are used to open the pathways of investigation. Although appearing 'messy', an inquiry unit of work can be scaffolded to provide opportunities for teachers to listen to and better understand their students as mathematicians. Recent work by Allmond, Wells and Makar (2010) describe inquiry as iterative cycles of investigating and reporting (4D Model) as students become engaged in an inquiry (Discover), plan how to answer the question (Devise), implement their plan of action (Develop) and reflect upon and justify their solutions (Defend). Although presented as phases, the 4D Model is not intended to be linear in nature and can be used by teachers as a planning tool to guide the direction of the inquiry.

Careful consideration of lesson sequencing or inquiry phases can foster occasions where mathematical understandings are challenged, making thinking explicit and visible. Students work together to solve inquiry problems by initially drawing on already developed conceptions which often differ to those of their peers. When these solutions are challenged by the task and other students, an active process of learning is fostered which values challenges and perturbations as students take the time to try and sort out the problem (DET, 2004). Further research into how and what students learn in an inquiry classroom can be used to refine understandings of this pedagogy and to offer teachers practical and useful strategies for teaching using this approach.

Learning

In this paper, the author has chosen an operational definition of learning used by Harel and Koichu (2010) to analyse conversations between the students and products of their work. This definition has previously been used to describe learning in a chapter devoted to the pedagogy of mathematical inquiry (Makar, 2012). Although seemingly complex and multi-dimensional itself, this definition "posits learning as a multi-dimensional and multi-phase phenomenon occurring when individuals attempt to solve what they view as a problem" (Harel & Koichu, 2010, p. 115). The definition supports constructivist views of learning evident in inquiry classrooms where multiple solutions to solving inquiry problems are also encouraged. The authors' view of learning is rooted in Piagetian theory which values phases of disequilibrium-equilibrium leading to assimilation and accommodation as part of the sequence of developing understanding, acknowledging the knowledge currently held and newly produced by students. The framework describes learning taking place when the dual relationship between perturbations and knowledge construction (*duality* -D) arise from the intellectual and psychological *necessity* (N) to develop knowledge or understanding. *Repeated-reasoning* (R) of the new ways of thinking and understanding developed assist the learner to internalise, organise and retain the new knowledge, described by Harel and Koichu as essential to the process of learning (DNR-based instruction).

Although the 2010 paper by Harel and Koichu tracks learning for one particular student, it does not provide detail regarding the source of learning descriptors, concerning both the mathematical content, ways of thinking, psychological and intellectual needs, etc. The author of this paper will rely on curriculum materials relevant to the context to illustrate student learning. The mathematical content explored in this inquiry was through the sub-strand (as identified by Australian Curriculum) of Location where students had opportunity to consolidate their understandings of creating and interpreting simple grid maps and showing position and pathways.

Also, whereas the illustration of the definition's operational nature provided in the 2010 paper highlights the researcher generating the perturbation for one participant (who is a mathematics teacher), this paper will highlight the importance of young students generating perturbations for themselves and others, making challenges more meaningful to themselves to solve. Harel and Koichu presented an analysis which demonstrated a strong theoretical background expanding on all elements of Burt's learning and you are encouraged to read this paper for further explication of all concepts discussed in this short summary of their work. The DNR-based definition will be useful to describe student learning in inquiry as it seems broad enough to encompass mathematical knowledge and problem-solving approaches that may exist in inquiry, as well as acknowledging learning as a continuum of phases congruent with learning mathematics though an inquiry approach.

Methodology

An inquiry mathematics classroom presents a complex, social environment where multiple variables may be measured and teaching and learning episodes may appear unpredictable, under little control by the researcher. The participants presented in this paper have been chosen to work together due to teaching and learning needs specific to the school environment, not to suit the researcher's requirements of random selection to compare to an appropriate comparison group. This presents difficulty if selecting a research methodology that values issues of external validity that can identify causal relations and rule out alternative explanations (Levin & O'Donnell, 2001). In qualitative research such as this study, it has become more accepted to choose research methods that fit the questions posed in the research (O'Donnell, 2005). This research uses Design Research (Cobb, et al., 2003) and forms part of a larger PhD study. The aim of the researcher-as-teacher is to contribute to the body of work that is developing a profile of inquiry in operation. The author has used their experience in teaching mathematics through inquiry to continue to 'engineer' this particular unit of work, in collaboration with their teaching partner. Although the unit, which questions 'Are maps useful?' has been developed in-line with the demands of the curriculum, the inquiry presents an opportunity to study learning in this context.

Participants and Context

The participants in this study involve a Year Three classroom of 24 students (ages 8 and 9) situated in a middle-class, technology-rich suburban primary school in Queensland. The learning described in this paper took place in the last term of the school year, where the students had experienced only three other units of work using this approach. This unit was based on lessons provided by the Queensland Department of Education, developed as a resource for the implementation of the Australian Curriculum. Mathematical content centred on location and requiring students to create and interpret simple grid maps showing positions and pathways.

Data Collection

Three students in particular have provided the focus for this paper; Dale, Brianna and Georgia (pseudonyms). These students were involved in a short conference with the teacher-as-researcher as they had finished creating a draft of their maps and were asked to reflect on these. This conference is common inquiry classrooms where students are encouraged to share successes and challenges with their peers, providing feedback that can be used for assessment purposes. The conference presented a conversation where it was possible to track and describe the learning taking place using an operational definition of learning. It was captured on an iPad using the camera's video function: firstly with the teacher present and then left with the students on record. The conversation was transcribed by the researcher and then analysed using the operational definition of learning, focusing on perturbations presented by the students. Products of these students' work were also collected including: collaborative brainstorms (see also Fry, 2011), maps the students had designed and created and written reflections about the unit and their own work conducted at the end of the one week unit.

Due to the short nature of this paper only Dale's learning will be illustrated using the operational definition. The other students have been included as they were involved in the conversations recorded and their input contributed to the development of map-related concepts such as gridlines and coordinates.

Analysis and Results

Data will be used to illustrate Dale's learning in the inquiry 'What makes the best map?" Conversations with and between students and products of the students' work were selected where they showed an emerging and developing understanding of the use and vocabulary concerning 'coordinates.' Observations of learning when students 'held' the moment where they didn't know what to do, generated by peers and considered 'perturbations' in this paper, are recorded as *ways of thinking* (*WoTh)* and *understanding* (*WoU)* about mapping and location using coordinates. These valuable moments were considered part of the learning process and are illustrated using Harel and Koichu's (2010) DNR-based definition of learning.

Collaborative brainstorming

Collaboration with peers is an essential part of an inquiry classroom environment. In activating and sharing prior knowledge there is a risk that students might feel that their ideas are not valued if not acknowledged by others. Early in this inquiry, collaborative brainstorming offered students opportunity to share what they already knew (Fry, 2011). Dale, Brianna and Georgia worked together throughout the inquiry. They were asked, "What is the best map? and encouraged to each record their ideas in writing on one shared sheet of paper, without any talk or collaboration of ideas. Dale wrote a detailed list of map features based on his prior knowledge of maps. He recalled maps in shopping centres he had visited e.g. "2A is like DFO" (A local outlet mall). His work indicated that he knew maps could answer questions such as "How to get there?" or "How many of something?" He did not use the mathematical word 'coordinates' or any ideas about how to make maps. After this opportunity the three students shared their ideas with each other and wrote map features that they agreed upon in collaboration in the centre of their page. Here the word 'coordinates' appears although nowhere else on the page.

Conversations

Dale: I put that on there. It's for coordinates so that you could say go to 3, D and you'll know where you are.
Teacher: Perfect, that's where Ian sits
Dale: But when you're in the classroom I was thinking you can't actually look if you don't have the map in front of you, so I was thinking...

Brianna interrupts: Because you wouldn't have exactly the same (map)... it says 4,D and you wouldn't know exactly where that is.

Teacher: Where's the 4 and where's the D? Is that what you mean?

Georgia: Yeah

Figure 1. Excerpt of conversation with students.

After a few days of viewing maps, creating their own individual maps, talking about them and sharing them with others, the students were asked again to re-join their original group to share their maps with each other. This was an opportunity for a teacher conference with one group of students resulting in two recorded conversations used as data in this paper. Although the conversation began with the teacher asking Dale about the features drawn on his map, he quickly noted the perturbation he had been considering, a problematic situation which he needed to solve (Figure 1). If a user did not have a map in front of them, what use would having coordinates be? He questioned the use of knowing the coordinates of a destination if you didn't have the map to show you. Georgia later presented her understanding that both users (the person giving instructions and the person looking for the destination) would need to have a copy of the map and that it would be have to be exactly the same. The three students agreed, content with the idea that coordinates were only useful when all users had the same map. The perturbation surfaced by the students could be said to lead to a motivation, or psychological need to solve the problem: contributing to the learning continuum of disequilibrium-equilibrium phases as described in the DNR-oriented definition of learning.

Shortly after this conversation the students were presented with a new task requiring continued collaboration, where they had to record similarities and differences between the maps now constructed. Even after agreement of the understanding of coordinates and that coordinates were useful only when all users had the same map, the students then disagreed about the use of coordinates on the maps they drew. Dale was certain that each map used coordinates as a useful feature. Again, Dale struggled with this concept of coordinates and what they were: he was still learning. Georgia quickly pointed out that although Brianna's map used gridlines, there were no letters down the side. Dale accepted this and decided to write the term 'coordinates' under the column titled 'Differences': all students agreed on what coordinates were and that not all three maps used coordinates.

Maps and Reflections

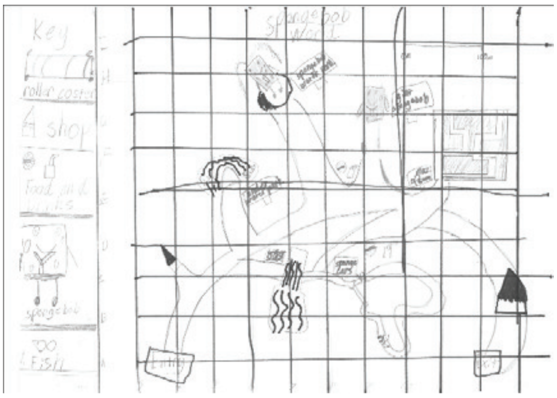

Figure 2. Dale's map: final product.

The nature of inquiry is that students Discover, Devise, Develop and Defend their ideas iteratively to solve the problem posed. This can result in many attempts by students to create what they think is 'the best map' for instance. When the Defend phase takes place at the end of the unit of work the students are encouraged to share their work to defend the use of the mathematics they chose. Features represented on Dale's map (Figure 2) included: a key, a graphic scale and gridlines marked with letters and numbers so that coordinates could be found. Dale noted in reflection that he used a grid "because if someone had the same map you could say go to 10C: you are at sponge cars." He continued to justify the usefulness of his map by explaining how coordinates assisted with direction, particularly when all users had the same map.

The collaborative brainstorming, the recorded conversation and the final product of student work show Dale's understandings about maps and mapping as something that occurred throughout the unit of work and not one point in time. Table 1 illustrates this using the DNR-oriented definition of learning in the Harel and Koichu (2010) paper.

Table 1
Illustration of Dale's learning: mathematical concepts that provided the focus of this paper were the uses of gridlines and coordinates in map making.

Ways of understanding		Collaborative Brainstorming	First conversation	Second conversation	Products of Student's work
	With respect to problem posing act:	Considers features of maps	If you don't have a map in front of you, what use are coordinates?	Similarities and differences between map features	Can successfully create a map
	With respect to the act of interpreting coordinates:	Combinations of letters and numbers are places on a map	Coordinates are only useful if all users have the same map	Grid lines are coordinates	Uses coordinates to show location on a map
	Ways of thinking:	Maps contain many features	Not all features on maps are useful	Grid lines show coordinates	Coordinates are useful on maps
		Conjecturing map features	Oral instructions do not rely on coordinates	All three maps use coordinates – this is a useful feature	My map is the best
	Psychological and intellectual needs:	The need to solve the problem *What is the best map?*	The need to know if you always need coordinates	The need for certainty	The need to justify choices made
	Ways of eliminating perturbations:	Collaboration with peers	Further investigation through discussion and use of maps made	Collaboration of ideas	Independently solving the task
	Missing components of the state of equilibrium:	Uncertainty about what is 'best'	Further practical investigation using maps	Clear definition of coordinates	

Discussion

Learning has been considered for one 8-year old student, Dale, as he attempted to solve through inquiry the problem "what is the best map?" His ways of understanding and thinking about location and mapping have been considered as influenced by two classroom peers of the same age, Brianna and Georgia. The operational definition used to illustrate learning in Harel and Koichu's (2010) paper relied on pe rturbations presented by the researcher, identified as the participant's psychological and intellectual needs. This research illustrates learning in an inquiry context promoted through the valuing of student-generated perturbations. Dale's understanding is presented as a continuum of disequilibrium-equilibrium phases (organised in this paper using the data) where he first considers features of maps he already knows, questions the usefulness of certain features and then finds equilibrium as he decides on particular mapping features that he thinks are best. In each phase he considers his peers' questions as feedback informing his own work.

He justifies his choices in the Defend phase of the inquiry: in this instance in written reflection. Possibly as Dale continues to see other students' presentations of their work he may again find himself perturbed by their choices. Such is the notion of learning as a continuum, congruent with learning in inquiry.

Dale formed his ideas about location and mapping in discussions with his teacher and peers, and possibly in discussions at home with his parents, shopping at his local shopping centre, playing with friends in the playground or reading further about the topic. The illustration of Dale's learning shows that through listening to and valuing students' conversations, generated perturbations can manifest intellectual and psychological needs that are meaningful to students and that engage them to solve the problem. Students learn to 'hold' onto the moment when they don't know the answer or what to do as a way of trying to sort out the problem.

The DNR-oriented definition of learning was developed by a group of researchers in work they conducted individually and together. In this paper, the researcher's description of the *WoU* and *WoTh* are subjective and were not reviewed in the same way by any other researchers. It would be useful for other researchers to review the literature used by the author and to analyse the data to compare results. *WofU* and *WoTh* are difficult to objectify and it was difficult to find a single framework that would provide consistent language to describe this. Further research refining the *WoU* and *WoTh* would allow other researchers to more easily use consistent terms in describing the learning of others. The operational definition of learning used in this paper shows promise for teachers to identify and assess student learning in inquiry.

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